Finally, we can obtain the pressure and temperature at the piston face from Eqs. (2) and (4) and the ideal gas equations. The results are

$$T/T_2 = [1+(\gamma+1)\tau/2]^{-2(\gamma-1)/(\gamma+1)} \tag{9}$$

and

$$P/P_2 = [1+(\gamma+1)\tau/2]^{-2\gamma/(\gamma+1)} \tag{10}$$

where T is the gas temperature.

Figures 3 and 4 show the trajectory of Z vs τ for $\gamma = 1.4$ and $\gamma = 1.66$ plotted against initial piston speed instead of N. As can be seen for large values of U the curves coalesce. This is just due to the fact that N approaches a maximum.

For very-high-speed models real gas effects must be included. This is most simply done by estimating the value of γ behind the shock. For air or easily excitable molecules, γ is as low as 1.1 - 1.15. However, for high-velocity models any oxygen bearing molecule will probably generate an excessive amount of free oxygen, which will react with the model. For deceleration using nitrogen as the stopping gas, γ will be close to 1.35 for realistic model speeds. Using the real shock conditions N can be found as well as P_2, T_2, and a_2 for a given set of initial catch tube pressure and temperature.

Two experimental points are available.[2] In the first, the point of reversal for a 46-g model was measured. Analysis indicates this point should lie near 29.7m, while the actual value is 28.0 m. The velocity for this measurement was 5.03 km/sec, and the gas was nitrogen. In the second piece of available data, the model did not reverse but was caught in a rag bundle after traversal of the tube. The initial velocity was 2.55 km/sec and the theory predicts a total stopping length of about 30 m. The model was traveling slowly enough to be easily captured in the rag bundle after a deceleration length of 33 m with no evident damage.

Care must be exercised in the comparison of data with the analysis since all the variables are sensitive to γ and the sound speed a_2. Values as close to actual as possible must be used to obtain good comparison.

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Correlation for the Viscosity of Air Including Effects of Dissociation

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A METHOD with which the viscosity of air can be simply and accurately calculated between a match point with the Sutherland formula at 1050°R and the enthalpy level, 20,000 Btu/1bm, at which ionization is about to become significant is presented here. The suggested expression, which is a function of temperature and the degree of dissociation, is a correlation of the results of Clark, et al.[1] That analysis of the transport properties of air is described briefly in the following paragraphs.

In the investigation of Ref. 1, the thermodynamic and transport properties of both air and pure nitrogen were calculated for pressures in the range 1-200 atm and temperatures in the range 1000-30,000K. This was done because available and well-know calculations[2-6] do not extend over the range of pressures and temperatures of interest, or they are based on outdated collision integrals. The present discussion is confined to the calculations for temperatures below 10,000K, which is the regime of interest in the present note. The thermodynamic properties calculated included the species mole fractions which are needed to compute the viscosity and other transport properties. The Aerotherm Chemical Equilibrium (ACE) computer program was used to calculate these species mole fractions, assuming the air system to be comprised of the species N_2, N, N^+, O_2, O, O^+, NO, NO^+, and e^-. The ACE calculations were compared with the tabulated results of Hilsenrath and Klein,[7] and were found to be within a few per cent at all times.

The transport properties were calculated using the mixture rules of Yos.[2] These expressions reduce to the results of rigorous kinetic theory in the limit of a one-specie gas. For mixtures, they are approximate in that they exclude the higher-order terms in the first Chapman-Enskog approximation. However, calculations based on the simpler mixture rules rarely differ from the more exact first approximation by more than a few per cent.[2] A literature review was conducted to determine the best set of collision integrals to use in conjunction with the Yos mixture expressions. The collision integrals presented in the original work of Yos were used whenever they were confirmed through the literature survey. However, a number of these integrals required updating (see Ref. 1 for details).

Using the formulation just described, we calculated the viscosity, and in addition, frozen and reactive thermal conductivity and electrical conductivity for air and pure nitrogen. The results of these calculations were compared critically with numerous experimental measurements of transport properties in the temperature range 6000-24,000K.[8-13] These data were acquired from experiments utilizing cascade electric arcs at 1 atm pressure. In general, the calculated transport properties agreed with the various measurements to within their associated experimental uncertainties. In addition, the present calculations were compared with several heavily referenced calculations available in the literature for the pressure range 1-100 atm.[2-6] In general, the present results and those of Peng and Pindroh[6] are in good agreement for all pressures and temperatures of interest. However, Hansen's[5] results, based on outdated collision integrals, deviate considerably from the present results.[1] In summary, it is felt that the calculations presented in Ref. 1 represent the best-validated and most current state-of-the-art air transport properties now available. They, therefore, provide a sufficiently firm foundation for the viscosity correlation presented herewith.

A somewhat different approximate kinetic theory analysis is employed in the Boundary Layer Integral Matrix Program (BLIMP code).[14-15] That analysis indicates that, for a mixture of fixed composition, the viscosity is well represented by a power law dependence upon temperature.

$$\mu/\mu_0 = (T/T_0)^{\omega}$$

This expression, with $\omega = 0.659$ (which is the value used in the BLIMP code) and with a modification which takes into account the effects of dissociation, is used here to correlate the air viscosities calculated by Clark.

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The power law representation may be matched at the low end of its range to the Sutherland expression for air viscosity, which is of the form

$$\mu/\mu_0 = (T/T_0)^{1.5}(T_0 + 198.6°\mathrm{R})/(T + 198.6°\mathrm{R})$$

Differentiation of the power law yields

$$\omega = (T/\mu)/(\mathrm{d}\mu/\mathrm{d}T)$$

For Sutherland viscosity

$$(T/\mu)(\mathrm{d}\mu/\mathrm{d}T) = 1.5 - T/(T + 198.6^0\mathrm{R})$$

With $\omega = 0.659$ the two expressions are matched at

$$T_0 = 1050°\mathrm{R}$$

where, from the Sutherland expression

$$\mu_0 = 1.992 \times 10^{-5}\ \mathrm{lbm/fps}$$

This suggests, for undissociated air at temperatures above 1050°R,

$$\mu = 1.992 \times 10^{-5}\ (T/1050°\mathrm{R})^{0.659}\ \mathrm{lbm/fps}$$

As shown in Fig. 1, this expression provides an excellent correlation of Clark's viscosity calculations except at temperatures high enough for dissociation.

Since the failure of the correlation at higher temperatures is associated with changes in composition, a modified correlation in terms of the degree of dissociation is suggested.

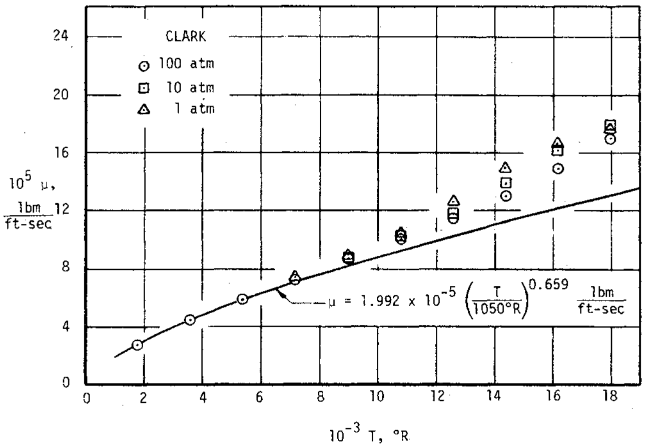

Fig. 1 Comparison of air viscosities calculated by Clark with the $\omega = 0.659$ power law expression.

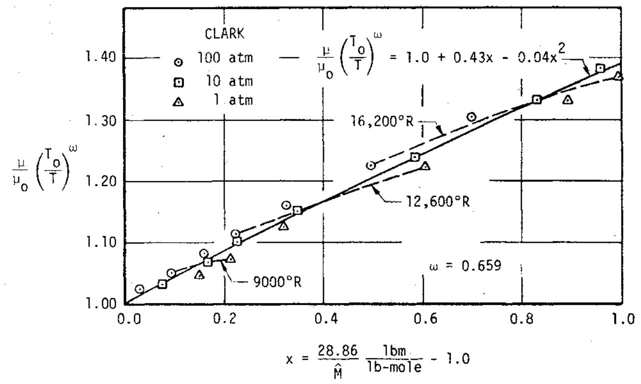

Fig. 2 Correlation of the effects of dissociation on air viscosity.

In Fig. 2, the departure of the viscosity from the power law expression is shown correlated in terms of the parameter

$$x = (28.86/\hat{M})/(\mathrm{lbm/lb\text{-}mole}) - 1.0$$

The maximum dissociation shown corresponds to an enthalpy of about 20,000 Btu/lbm. The power law modified for the effects of dissociation becomes

$$\mu = 1.992 \times 10^{-5}\ (1.0 + 0.43x$$
$$- 0.04x^2)\ (T/1050°\mathrm{R})^{0.659}\mathrm{lbm/fps}$$

This expression is accurate (that is, it correlates Clark's viscosities) to within ±2% in the range from 1050°R to the 20,000 Btu/lbm enthalpy level.

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